Interpolated Controllers for the Robust Transition Control of a Class of Reactors

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Two new theoretical results about interpolated controllers for the transition control and simultaneous stabilization of single input-single output plants with variable operating conditions are shown; recent results are generalized for interpolated controllers. On the basis of these results, an application to the closed-loop transition control of a continuous stirred-tank reactor, where a single exothermic reaction takes place, is presented. A comparison with simple PI controllers is made. © 2005 American Institute of Chemical Engineers *AIChE J,* 52: 247–254, 2006
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Introduction

During plant operation, variations in operating conditions may occur frequently. For example, in polymerization plants, grade transition operations are normally carried out when polymer properties are modified to reflect the production of new polymer materials.[1,2] Such grade transition operations consist of predetermined setpoint changes. Batch operation, startup and shutdown of continuous equipment, are also industrial examples where processing conditions might vary widely. Even if the plant is located within the nominal design region, process upsets might hit the system so that excursions away from the nominal operating region might be expected. The existence of chemical processes with wide variations in operating conditions may impose some control problems since closed-loop control systems are normally designed, based on information about nominal design conditions. Therefore, controller performance might degrade if process conditions are radically different from the nominal ones.

Some approaches suggested to cope with the closed-loop transition control of plants with wide variations in process dynamics could be classified as control structures that use a control law and an observer scheme. Under this category, adaptive control techniques,[3] and model predictive controllers[4] are embedded. Such control schemes are attractive, but they might be difficult to design, implement and maintain due to complexities in the controller and observer design. A more simple control technique involves the use of the so-called gain-scheduling controllers. In this technique a family of plants is derived (that is, around a tracking reference signal), and a set of controllers is designed for each plant within the set of family plants. A switching strategy is used to decide which controller should be used. Using this approach simple PI controllers could be used to cope with the closed-loop control of plants where wide variations in process conditions and plant dynamics are expected.

It has been reported that up to 90–95% of the industrial controllers are of PID type.[5] This is so because PID controllers are easy to design and easy to tune by plant personnel. Industrially, these controllers are often tuned by trial and error procedures, based on heuristics and on plant experience; if process conditions change drastically, such controllers may need retuning. However, it is a well known fact that PID based control systems do not perform well for plants with process gains that change sign among operating regions. Also, if controller parameters are fixed, these types of feedback systems may not be able to cope with wide variations in process dynamics. There are some ways to improve simple PID control if some kind of automatic tuning, gain scheduling or continuous adaptation is incorporated into the design process.[5]

One the main objectives of the analysis tool described in this paper is to provide analysis results, and make some comments regarding the control law. In addition, when considering interpolated control standard techniques of gain-scheduling or interpolated control might be used. Accordingly, the problem of simultaneous robust stabilization of a structured multiparameterized family of SISO linear plants is addressed. Two new results about interpolated controllers for plants with variable operating conditions are presented, these results generalize recent results by [6]. For this purpose, a set of interpolating controllers is designed. In this approach an "interpolated" controller, is considered as a linear interpolation of coprime factorizations of stabilizing controllers for the representative models as described in [7]. In addition, the results provide sufficient conditions for a controller to be an interpolated controller which simultaneously stabilizes the family of plants. One of these results is based in sums of Hurwitz polynomials, the other result uses units in the set of SPR0 (strictly positive real of relative zero degree) functions. This approach is useful for the synthesis of controllers that stabilize linear parameter-varying systems in which the parameter is piecewise constant and does not change often. There exist several examples of practical systems, which can be modelled in this form.[6,8,9] Another important feature of interpolated controllers refers to failure tolerant characteristics, meaning that if one or several controllers break down the stabilization of the interpolated plant is guaranteed as long as least one controller works properly.

This article outlines as follows. In the first section some preliminary concepts to prove closed-loop stability of the interpolated controller are given. In the second section, the stabilization of a linear SISO interpolated plant with variable operating conditions using an interpolated controller is considered. In the third section, the interpolated control approach is applied to the transition control of a continuous well stirred tank reactor. This section also contains the interpolated plant and control representation, tuning of the interpolated controller and discussion of the results obtained in this work. The fourth section contains the conclusions of this article. The appendix section contains the mathematical proofs of the two main results of our work.

Preliminaries

In this section definitions and notation necessary for the theoretical part of this work are presented.

Let RH^∞ be the set of the proper, stable, and rational real functions. $U(RH^\infty)$ is the set of the units in RH^∞, that is, invertible elements in the set of units RH^∞. The following fact is used in this article.

Fact 1. *The following two statements are equivalent:*

1. *The convex combination of two Hurwitz polynomials* $p(s)$ *and* $q(s)$

$$\lambda p(s) + (1 - \lambda) q(s) \quad for\ \lambda \in [0, 1]$$

is a Hurwitz polynomial.

2. *The convex cone of two Hurwitz polynomials* $p(s)$ *and* $q(s)$

$$ap(s) + bq(s) \quad for\ a\ and\ b\ positive\ reals$$

is a Hurwitz polynomial.

The earlier fact is consequence of the following substitution

$$\lambda = \frac{a}{a+b} \quad \text{and} \quad (1-\lambda) = \frac{b}{a+b}$$

note that fact 1, can be generalized for the convex combination of n Hurwitz polynomials. In the rest of the article we use the notation a and b. It is important to observe that this equivalence does not mean that the stability of two polynomials imply the stability of the convex combination; it only establishes the equivalence between the two statements. Recall that any real polynomial $p(s)$ can be represented as

$$p(s) = p^e(s^2) + sp^o(s^2) \tag{1}$$

where $p^e(s^2)$ is the even part of $p(s)$ and $p^o(s^2)$ is the odd part of $p(s)$. In order to discuss next results the following plant families are defined.

The family given by

$$F_{1T} \equiv \left\{ g(s) : g(s) = \frac{a_1 N_1(s) + \cdots + a_T N_T(s)}{a_1 D_1(s) + \cdots + a_T D_T(s)} \right\} \tag{2}$$

$$p_i(s) = \frac{N_i(s)}{D_i(s)} \quad \text{for } i = 1, \ldots, T \tag{3}$$

where $N_i(s)$ and $D_i(s)$ are real polynomials, $\deg[a_1 D_1(s) + \cdots + a_T D_T(s)] = n$, $\forall\ a_1, \ldots, a_T \in \mathbb{R}^+ - \{0\}$ or $\forall\ a_1, \ldots, a_T \in \mathbb{R}^- - \{0\}$ and $\deg N_i(s) = \deg D_i(s) = m$ for $i = 1, \ldots, T$.

Stabilization by Interpolated Controllers

In this section, the stabilization of a linear time-invariant single-input/single-output interpolated plant with variable operating conditions using an interpolated controller, in a similar way to [6], is addressed. This section also contains the two main theoretical results of this work. The interpolated controller design task is also addressed in this section. The formulation of the problem for the first result is as follows.

Let us take the closed-loop system of a plant $g_{ab}(s)$ and a controller $c_{xy}(s)$

$$\frac{g_{ab}(s)}{1 + c_{xy}(s)\, g_{ab}(s)} \tag{4}$$

Let $g_1(s) = [N_1(s)]/[D_1(s)]$ and $g_2(s) = [N_2(s)]/[D_2(s)]$ be two representative transfer function models of $g_{ab}(s)$, which are defined at two different operating points.

(a) For an operating point located somewhere between the representative ones, the transfer function of the plant $g_{ab}(s)$ is described by a linear "interpolation" of the polynomials numerator and denominator as $g_{ab}(s) = N(s)D^{-1}(s)$ with

$$N(s) = aN_1(s) + bN_2(s) \tag{5}$$

$$D(s) = aD_1(s) + bD_2(s) \tag{6}$$

where a, $b \in \mathbb{R}$ are two parameters which represent the change of plant dynamics such that a, b are not both simultaneously equal to zero.

(b) The operating conditions do not change so often such that the parameters a, b can be identified from some information about operating conditions.

A controller which can cope with plant dynamics variations is now introduced. Let $c_1(s) = [N_{c1}(s)]/[D_{c1}(s)]$ and $c_2(s) = [N_{c2}(s)]/[D_{c2}(s)]$ be two stabilizing controllers for the representative models $g_1(s)$ and $g_2(s)$, respectively. Consider an interpolated controller $c_{xy}(s) = N_c(s)D_c^{-1}(s)$, where

$$N_c(s) = xN_{c1}(s) + yN_{c2}(s) \tag{7}$$

$$D_c(s) = xD_{c1}(s) + yD_{c2}(s) \tag{8}$$

x, $y \in \mathbb{R}$ are two parameters which represent the change of controller dynamics such that x, y are not both simultaneously equal to zero.

The objective of this section is to present sufficient theoretical conditions such that the interpolated controller $c_{xy}(s)$ will be able to stabilize the interpolated plant $g_{ab}(s)$ for a, b. The results obtained are Kharitonov-like stability for interpolated controllers. In addition, the interpolated controller $c_{xy}(s)$ (family of controllers) simultaneously stabilizes each plant in F_{12}. We now present the first stability result about interpolated controllers.

Proposition 1. *The interpolated controller*

$$c_{xy}(s) = \frac{xN_{c1}(s) + yN_{c2}(s)}{xD_{c1}(s) + yD_{c2}(s)}$$

stabilizes the family of plants F_{12} $\forall\ a, b, x, y \in \mathbb{R}^+$ *or* $\forall\ a, b, x, y \in \mathbb{R}^-$ *where the* a, b, x, y *parameters are not simultaneously equal to zero. If the following* 16 *polynomials:* $q_i^e(s^2) + sq_j^o(s^2)$ *are Hurwitz for* $i, j = 1, 2, 3, 4$ *with, and* $\deg q_i(s) = m$, *that is, the order of the polynomials are invariant for* $i = 1, 2, 3, 4$, *where*

$$N_1(s)N_{c1}(s) + D_1(s)D_{c1}(s) = q_1^e(s^2) + sq_1^o(s^2) = q_1(s)$$

$$N_2(s)N_{c2}(s) + D_2(s)D_{c2}(s) = q_2^e(s^2) + sq_2^o(s^2) = q_2(s)$$

$$N_2(s)N_{c1}(s) + D_2(s)D_{c1}(s) = q_3^e(s^2) + sq_3^o(s^2) = q_3(s)$$

$$N_1(s)N_{c2}(s) + D_1(s)D_{c2}(s) = q_4^e(s^2) + sq_4^o(s^2) = q_4(s)$$

(*For the proof see the Appendix*).

Using Fact 1, this result can be interpreted as if the interpolated controller given by

$$c_\alpha(s) = \frac{\alpha N_{c1}(s) + (1-\alpha)N_{c2}(s)}{\alpha D_{c1}(s) + (1-\alpha)D_{c2}(s)} \tag{9}$$

would be able to stabilize the interpolated plant

$$g_\lambda(s) = \frac{\lambda N_1(s) + (1-\lambda)N_2(s)}{\lambda D_1(s) + (1-\lambda)D_2(s)} \tag{10}$$

for all α, $\lambda \in [0, 1]$, providing the conditions of proposition 1 are met. Therefore, proposition 1 is stronger than the results stated in [6], because in this case it is possible that $\alpha \neq \lambda$, and more important, each controller in the family $c_{xy}(s)$, simultaneously stabilizes all family of plants F_{12}. For itself this property can be understood as a fault tolerant control characteristic, meaning that if some or several controllers break down, but at least one of them works correctly, the stabilization of the interpolated plant is guaranteed. But in our result, proper stable coprime factorizations of the plants were not used. This fact motivates us to present a result that uses proper stable coprime factorizations, that will be stated in proposition 2.

Note that using a similar proof of proposition 1 (see the Appendix), this result for the family F_{1T} and the interpolated controller

$$c(s) = \frac{a_1N_{c1}(s) + \cdots + a_TN_{cT}(s)}{a_1D_{c1}(s) + \cdots + a_TD_{cT}(s)} \tag{11}$$

can be generalized. The formulation of the problem for the second result is the same, but in this case the F_2 family will be given by

$$F_2 \equiv \left\{ g_{ab}(s) : g_{ab}(s) = \frac{an_1(s) + bn_2(s)}{ad_1(s) + bd_2(s)} \right\} \tag{12}$$

with a, $b \in R^+$ or a, $b \in R^-$ where the a, b parameters are not simultaneously equal to zero, with $n_1, n_2, d_1, d_2 \in RH^\infty$; n_1, d_1 is a proper stable coprime factorization of $g_1(s) = [n_1(s)/d_1(s)]$, while n_2, d_2 is a proper stable coprime factorization of $g_2(s) = [n_2(s)]/[d_2(s)]$, and $\deg[ad_1(s) + bd_2(s)] = m$. The interpolated controller is given by

$$\bar{c}_{uv}(s) = \frac{un_{c1}(s) + vn_{c2}(s)}{ud_{c1}(s) + vd_{c2}(s)} \tag{13}$$

with u, $v \in R^+$ or u, $v \in R^-$, where the u, v parameters are not simultaneously equal to zero with

$$n_{c1} = x_1 + r_1d_1, \quad d_{c1} = y_1 - r_1n_1$$

$$n_{c2} = x_2 + r_2d_2, \quad d_{c2} = y_2 - r_2n_2$$

Proposition 2. *The interpolated controller* $\bar{c}_{uv}(s)$ *stabilizes the family of plants* F_2; $\forall\ a, b, u, v \in R^+$ *or* $\forall\ a, b, u, v \in R^-$, *where the* a, b, u, v *parameters are not simultaneously equal to zero with*

$$x_1n_1 + y_1d_1 = 1 \tag{14}$$

$$x_2n_2 + y_2d_2 = 1 \quad (15)$$

if the following condition is met:

(1) *There exist* $r_1, r_2 \in RH^\infty$, *such that the following two rational functions*

$$n_2x_1 + d_2y_1 + r_1(d_1n_2 - n_1d_2) \quad (16)$$

$$n_1x_2 + d_1y_2 + r_2(d_2n_1 - n_2d_1) \quad (17)$$

are strict positive real functions of zero relative degree (*For the proof see the Appendix*).

Note that by theorem 5.4.2 in [7] and since Eqs. 16 and 17 are units, both plants $\bar{g}_1(s) = [n_1(s)]/[d_1(s)]$ and $\bar{g}_2(s) = [n_2(s)]/[d_2(s)]$ are simultaneously stabilized by each one of the controllers $\bar{c}_1(s) = [n_{c1}(s)]/[d_{c1}(s)]$ and $\bar{c}_2(s) = [n_{c2}(s)]/[d_{c2}(s)]$.

Using fact 1, the proposition 2 can be interpreted exactly as in the results shown by [6], furthermore each controller in the interpolated controller, simultaneously stabilizes all the family F_2. The same comment stated for proposition 1 regarding fault tolerant control characteristics is valid for this proposition as well.

It is worth mentioning that both propositions 1 and 2 establish proofs of simultaneous stabilization of all family of plants generated by the interpolated plants for each one of the controllers obtained from the interpolated controllers. The main difference between propositions 1 and 2 lies in the fact that, while in proposition 1 the set of plants and controllers are described in terms of a quotient of polynomials, in proposition 2 the same set of plants and controllers are represented in terms of proper and stable rational functions in RH^∞, that is, in terms of rational coprime factorizations in the sense of [7].

It is important to stress that one way of using our results is the following one: If $\lambda \neq \alpha$ then the controllers are able to simultaneously stabilize the plants for frozen values of the uncertain parameter. On the other hand, when $\lambda = \alpha$ interpolated controllers should be used. In the last case the parameters of the controller might be adapted dynamically as a function of the plant parameters. In this way this sort of control strategy becomes general compared to control strategies.[6,10,11]

We must stress that the implications from proposition 2 are much stronger than those reported in [6]. In fact, using proposition 2 we might be able to stabilize *any* plant, generated by interpolating plant description, with *any* controller obtained from interpolating controller description. On the other hand, the results obtained in [6] are more restrictive: a certain controller can only stabilize the plant for which it was synthesized (that is, such controller is unable to stabilize *any* plant generated by plant description). This is the main theoretical contribution of this work. The results from propositions 1 and 2 are relatively easily extended to the case of n plants and controllers. Notice that the number of polynomials to check is around n with exponent 4, where n is the number of systems. For this reason, this method is suitable for at most 4 systems.

SISO Control of an Exothermic Reactor

In this section the tracking closed-loop control of a continuous stirred tank reactor (CSTR), where the single exothermic reaction $A \rightarrow B$ takes place, is addressed. The dynamic model of the CSTR is described by the following set of nonlinear ordinary differential equations[11]

Table 1. Nominal Parameter Values

Volumetric feed flow rate	q	100	lt/min
Feed concentration	C_{Af}	1	mol/lt
Feed temperature	T_f	350	°K
Inlet coolant temperature	T_{cf}	350	°K
Volume	V	100	lt
Heat-transfer term	h_A	7.5×10^5	cal/(min-°K)
Preexponential factor	k_o	7.2×10^{10}	1/min
Activation energy term	E/R	1×10^4	°K
Heat of reaction	ΔH_r	-2×10^5	cal/mol
Reaction mixture density	ρ	1×10^3	gr/lt
Coolant density	ρ_c	1×10^3	gr/lt
Reaction mixture specific heat	C_p	1	cal/(gr-°K)
Coolant specific heat	C_{pc}	1	cal/(gr-°K)

$$\frac{dC_A}{dt} = \frac{q}{V}(C_{Af} - C_A) - k_o e^{-E/RT} \quad (18)$$

$$\frac{dT}{dt} = \frac{q}{V}(T_f - T) + k_1 C_A e^{-E/RT} + k_2 q_c(1 - e^{-k_3/q_c})(T_{cf} - T) \quad (19)$$

where

$$k_1 = \frac{-\Delta H_r k_o}{\rho C_p} \quad (20)$$

$$k_2 = \frac{\rho_c C_{pc}}{\rho C_p V} \quad (21)$$

$$k_3 = \frac{h_A}{\rho_c C_{pc}} \quad (22)$$

where C_A and T stand for product concentration and reactor temperature, respectively. The parameter meaning and nominal design values are shown in Table 1.

The control objective of the CSTR consists in the transition control of the product composition (C_A) using the coolant flow rate (q_c) as the manipulated variable. For this purpose the nonlinear plant was linearized around three nominal steady-states. The operating conditions and transfer function information for each design point are shown in Table 2. Simple PI controllers were designed for each operating point; controller settings are also shown in Table 2.

Interpolated Plant and Controller

In this work an analysis method is presented and some hints about the design of the controller are sketched. When the vertex controllers do not satisfy the sufficient condition for interpolation, this method is not suitable and other interpolation method is necessary.

The resulting interpolated plant and controller are now derived. For the case of three plants the following characteristic polynomial must be Hurwitz

Table 2. Operating Regions Design Values, Transfer Functions, and PI Controller Settings

Operating Region	C_A mol/lt	T °K	q_c lt/min	$C_A(s)/q_c(s)$ (mol-min)/lt^2	Poles Location	k_c mol/lt-°K	τ_I min
1	0.08	441.27	99.93	$\frac{.0386}{s^2 + 4.8458s + 14.1904}$	$-2.43 \pm 2.8844i$	119.43	0.6
2	0.1	438.54	103.41	$\frac{.0411}{s^2 + 2.6735s + 10.9675}$	$-1.3367 \pm 3.03i$	65.18	0.2439
3	0.12	434.63	108.1	$\frac{.0389}{s^2 + 1.0969s + 8.0074}$	$-0.5484 \pm 2.7761i$	27.93	0.3

$$P_{ab}(s) = (aN_1 + bN_2 + cN_3)(xN_{c1} + yN_{c2} + zN_{c3}) + (aD_1 + bD_2 + cD_2)(xD_{c1} + yD_{c2} + zD_{c3}) \quad (23)$$

where the plants are given by

$$g_1(s) = \frac{N_1(s)}{D_1(s)} \quad (24)$$

$$g_2(s) = \frac{N_2(s)}{D_2(s)} \quad (25)$$

$$g_3(s) = \frac{N_3(s)}{D_3(s)} \quad (26)$$

for the above set of plants simple PI controllers

$$c_i(s) = \frac{N_{ci}(s)}{D_{ci}(s)} = \frac{k_{i1}s + k_{i2}}{s} = k_{i1}\left(1 + \frac{1}{\tau_i s}\right), \quad i = 1, 2, 3 \quad (27)$$

were tuned. Hence, using the plant transfer function descriptions and the controller settings shown in Table 1, the following polynomials, and the 81 polynomials resultants of the combinations of their even and odd parts, must be Hurwitz

$$q_1(s) = N_1N_{c1} + D_1D_{c1} = s^3 + 4.8458s^2 + (0.0386k_{11} + 14.19)s + 0.0386k_{12} \quad (28)$$

$$q_2(s) = N_1N_{2c} + D_1D_{2c} = s^3 + 4.8458s^2 + (0.0386k_{21} + 14.19)s + 0.0386k_{22} \quad (29)$$

$$q_3(s) = N_1N_{3c} + D_1D_{3c} = s^3 + 4.8458s^2 + (0.0386k_{31} + 14.19)s + 0.0386k_{32} \quad (30)$$

$$q_4(s) = N_2N_{c1} + D_2D_{c1} = s^3 + 2.6735s^2 + (0.0411k_{11} + 10.968)s + 0.0411k_{12} \quad (31)$$

$$q_5(s) = N_2N_{2c} + D_2D_{2c} = s^3 + 2.6735s^2 + (0.0411k_{21} + 10.968)s + 0.0411k_{22} \quad (32)$$

$$q_6(s) = N_2N_{3c} + D_2D_{3c} = s^3 + 2.6735s^2 + (0.0411k_{31} + 10.968)s + 0.0411k_{32} \quad (33)$$

$$q_7(s) = N_3N_{c1} + D_3D_{c1} = s^3 + 1.0969s^2 + (0.0389k_{11} + 8.0074)s + 0.0389k_{12} \quad (34)$$

$$q_8(s) = N_3N_{2c} + D_3D_{2c} = s^3 + 1.0969s^2 + (0.0389k_{21} + 8.0074)s + 0.0389k_{22} \quad (35)$$

$$q_9(s) = N_3N_{3c} + D_3D_{3c} = s^3 + 1.0969s^2 + (0.0389k_{31} + 8.0074)s + 0.0389k_{32} \quad (36)$$

A sufficient condition for all polynomials to be Hurwitz, is that the following inequalities are met

$$1.0969(0.0389k_{i1} + 8.0074) > 0.0411k_{i2} \quad \text{and} \quad k_{i1}, k_{i2} > 0 \quad \text{for } i = 1, 2, 3$$

In this case, if $\psi(k_{i1}) = 1.0969(0.0389k_{i1} + 8.0074)$ and $\phi(k_{i2}) = 0.0411k_{i2}$, then

$$\psi(k_{11}) = 13.879 > \phi(k_{12}) = 8.1809$$
$$\psi(k_{21}) = 11.565 > \phi(k_{22}) = 10.9834$$
$$\psi(k_{31}) = 9.9751 > \phi(k_{32}) = 3.8264$$

Therefore, the interpolated controller

$$c_{xy}(s) = \frac{x(119.43s + 199.05) + y(65.18s + 267.24) + z(27.93s + 93.1)}{(x + y + z)s} \quad (37)$$

simultaneously stabilizes the plants given by

$$g_{ab}(s) = \frac{0.0386a + 0.0411b + 0.0389c}{a(s^2 + 4.8458s + 14.1904) + b(s^2 + 2.6735s + 10.9675) + c(s^2 + 1.0969s + 8.0074)} \quad (38)$$

Table 3. Initial and Final Transition Points and PI Controller Settings for Transition Control Between the Operating Regions

Transition	Initial	Final	U_l	t_l	t_u	K_{c1}	τ_{I1}	k_{c2}	τ_{I2}	K_{c2}	τ_{I3}
1	0.08	0.1	0.083	20	100	119.43	0.191	65.9	0.18	27.93	0.2987
2	0.1	0.12	0.11	10	100	119.43	0.3302	200	0.05	100	0.7
3	0.12	0.1	0.1	40	100	119.43	0.3344	100	0.25	27.93	0.3
4	0.12	0.08	0.08	1	100	119.43	0.3344	65.91	0.2439	27.93	0.3

for all pair $(a, b, c) \in R^3$ such that $a, b, c \geq 0$, and if $a = 0$ then $b \neq 0$ or $c \neq 0$, if $b = 0$ then $a \neq 0$ or $c \neq 0$, and if $c = 0$ then $a \neq 0$ or $b \neq 0$.

It remains as a task to specify the way both plant and controller interpolation is done; notice that this is the aim of the x, y, z and a, b, c coefficients in Eqs. 37 and 38, respectively.

For the problem at hand a more general approach for setting both the interpolated plant and controller consists in forming the global dynamic model $[G(\gamma)]$ by the combination of the three local linear plants

$$G(\gamma) = \frac{\gamma_1 N_{G1}(s) + \gamma_2 N_{G2}(s) + \gamma_3 N_{G3}(s)}{\gamma_1 D_{G1}(s) + \gamma_2 D_{G2}(s) + \gamma_3 D_{G3}(s)} \tag{39}$$

similarly the global control behavior will be given by the interpolated response from the three local linear controllers

$$K(\gamma) = \frac{\gamma_1 N_{K1}(s) + \gamma_2 N_{K2}(s) + \gamma_3 N_{K3}(s)}{\gamma_1 D_{K1}(s) + \gamma_2 D_{K2}(s) + \gamma_3 D_{K3}(s)} \tag{40}$$

This approach is more practical when testing different PI controller settings since the algebraic procedure leading to Eqs. 37 and 38 could be tedious and prone to errors.

Switching Policy and Tuning

When a plant operates somewhere between two operating regions, plant behavior is dictated by a combination of two adjacent transfer functions; the computation of the global plant and control behavior is the aim of the so-called interpolating functions. Such functions are used to specify the way transitions among plant and controller descriptions are done. The desired transitions should be enough smooth so that hard discontinuities and strong control actions are avoided. In this work, the interpolating functions (γ_i) description were taken as those proposed in [11]

$$\gamma_1 = \begin{cases} e^{-(C_A-0.08)^2/0.008^2} & \forall\ C_A \in [0.08, 0.1] \\ 0 & \forall\ C_A \in [0.1, 0.12] \end{cases} \tag{41}$$

$$\gamma_3 = \begin{cases} 0 & \forall\ C_A \in [0.08, 0.1] \\ 1 - e^{-(C_A-0.1)^2/0.008^2} & \forall\ C_A \in [0.1, 0.12] \end{cases} \tag{42}$$

$$\gamma_2 = \begin{cases} 1 - \gamma_1 & \forall\ C_A \in [0.08, 0.1] \\ 1 - \gamma_3 & \forall\ C_A \in [0.1, 0.12] \end{cases} \tag{43}$$

notice that in Eqs. 37 and 38 $x = a = \gamma_1$, $y = b = \gamma_2$, and $z = c = \gamma_3$, respectively.

In order to compute the best tuning parameters for transition control of the set of 3 PI controllers, the following minimax problem was solved

$$\min_{K_{c1}, \tau_{I1}, K_{c2}, \tau_{I2}, K_{c3}, \tau_{I3}} (\mathrm{Max}\{y_{out}\})$$

s.t.

$$y_{out} \geq U_l \quad \forall\ [t_l, t_u] \tag{44}$$

where y_{out} stands for plant response in deviation form. Using this optimization tuning approach the maximum value of the plant response is minimized for any time. However, minimizing the maximum plant response might lead to closed-loop responses far from the desired setpoint. To keep the plant response close enough to the setpoint a constraint is added that specifies that y_{out} should be greater than a minimum bound (U_l) for all the times between lower and upper bounds denoted by t_l and t_u, respectively. In consequence, the Min Max optimization problem may result in a local optimal solution. Table 3 contains a summary of all the results of the tuning computations done by using the Matlab optimization toolbox,[12] and the Matlab/Simulink connection facility for dynamic simulation. The potential stability problem due to additional unmeasured disturbances can be approached using standard observer techniques for parameters estimation. This problem will be addressed in a future work.

Results

In this section, closed-loop results on the transition control around the defined nominal operating regions are shown; in order to obtain more realistic results small amounts of noise were added to the output measured control signals. In all the following closed-loop response figures "Interpolating Control" means results obtained using our interpolated control approach, while "Local Control" means simple PI control applied only to the plant associated to the initial transition point. Initial and ending transition points are shown in Table 3.

Regarding transition 1 (see Figure 1) notice that when using the interpolated controller, a slightly oscillatory response is observed damped as the final setpoint is approached. The control effort is not so demanding partially due to the small setpoint change. Note that the required control actions are different depending whether an interpolated plant description or the corresponding nominal one is used; this behavior is perfectly valid because different plant representations are used by each controller. Using an interpolated plant and control descriptions transition 2 shows a more pronounced oscillatory response. In comparison, using the corresponding plant representation a smoother transition is observed. However, the behavior obtained using the interpolated controller must be closer to the real expected situation simply because it uses more information about the system to be controlled. Again, different final control actions are observed for each controller. This situation can be explained on the same ground as transition 1

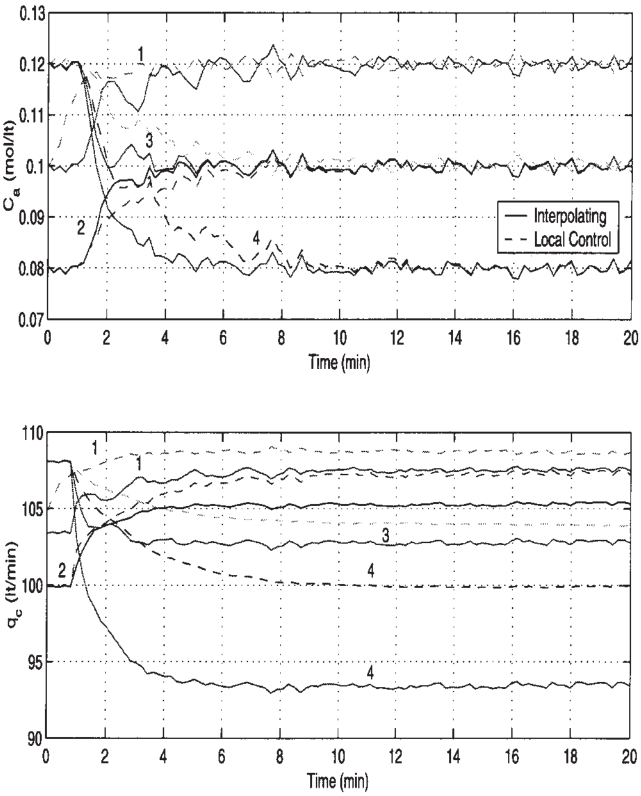

Figure 1. Closed-loop tracking behavior for all the transitions, numbers refer to product transitions as defined in Table 3.

was. Using an interpolated plant and controller description transition 3 is not so difficult to carry out as observed in Figure 1. In fact, it is faster compared to the case of a single plant. This is partially due to the interpolated nature of the control system but also due to the intrinsic dynamic plant behavior as explained later on. With respect to transition 4 if an interpolated controller is used it turns out that this transition is the simplest one to carry out. Note that, contrary to transitions 1 and 2, the local control shows oscillatory behavior as it also did in transition 3. Again, the difference in the final value of the control action can be explained as we did in transition 1.

As noted from the earlier results, using interpolated plant and controller descriptions transitions 1 and 2 were more difficult to carry out in comparison to transitions 3 and 4. This behavior is easy to explain if we recall, from Table 2, the poles location for each nominal operating regions. In fact, we observe that the imaginary part of the poles is almost the same independently of the operating region; however, the poles location differ mostly in their real parts. Operating region 1 has a faster constant time compared to operating region 3. Therefore, due to the magnitude of the real and imaginary parts of the poles location, region 3 must show stronger oscillatory behavior. Finally, the reason why the manipulated variables take different values when doing a closed-loop transition computation is due to the fact that the closed-loop simulations were done using transfer functions representation of the plant dynamics. Because we use a single plant transfer function for "traditional" SISO PI controllers, and three plant transfer functions (depending upon the operating range) for interpolating controllers, the final values of the manipulated variables in those cases will not match exactly. Had we used the nonlinear model representation of plant dynamics, the final value of the manipulated variables should be the same.

Conclusions

We give two results about interpolated controllers for plants with variable operating conditions, these results generalize recent results for interpolated controllers and families of controllers, where each one simultaneously stabilizes a certain family of plants. On the basis of this result an application to the closed-loop tracking control of a CSTR, where a single exothermic reaction takes place, was presented. Using the interpolated controller approach better control was obtained for more difficult to operate regions.

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Appendix

In this appendix we give some necessary results to prove the results in the previous sections.

Definition 1. *The Bezoutian,* $Bez[p(x), q(y)]$, *of two polynomials* $p(s)$ *and* $q(s)$[13,14] *is the symmetric matrix* $B_{i,j}$ *where*

$$\frac{p(x) - q_{(y)}}{x-y} = \sum_{i,j} B_{ij} x^i y^j \tag{45}$$

*Lemma A*1. *Let* $p_i(s) = {}_ip_m s^m + \cdots + {}_ip_0$ *be polynomials with* ${}_ip_m > 0$ *for* $i = 1, \ldots, T$, *such that* $p_i^e(s^2) + sp_j^o(s^2)$

are Hurwitz polynomials for $i, j = 1, \ldots, T$, *where* $p_i(s) = p_i^e(s^2) + sp_i^o(s^2)$ *are the decompositions in even and odd part of* $p_i(s)$. *Then* $\sum_{i=1}^{T} a_i p_i(s)$ *is a Hurwitz polynomial for* $a_i \in \mathbb{R}$ *such that either* $a_i > 0$ *or* $a_i < 0$.

The above Lemma can also be proved using the results in [15] or in [10].

*Proof of Lemma A*1. Note that

$$\sum_{i=1}^{T} a_i p_i(s) = \sum_{i=1}^{T} a_i p_i^e(s^2) + s\left[\sum_{i=1}^{T} a_i p_i^o(s^2)\right]$$

Computing the Bezoutian and using lemma A2 from this appendix

$$Bez\left[\sum_{i=1}^{T} a_i p_i^e(s), \sum_{j=1}^{T} a_j p_j^o(s)\right] = \sum_{i=1}^{T}\sum_{j=1}^{T} a_i a_j Bez[p_i^e(s), p_j^o(s)].$$

By hypothesis $p_i(s) = p_i^e(s^2) + sp_i^o(s^2)$ and $p_i^e(s^2) + sp_j^o(s^2)$ are Hurwitz polynomials for $i, j = 1, \ldots, T$ and $sgn(a_1) = \cdots = sgn(a_T)$. Then by lemma A3 from this appendix

$$Bez[p_i^e(s), p_j^o(s)] > 0$$

for $i, j = 1, \ldots, T$. In consequence $\sum_{i=1}^{T} a_i p_i(s)$ is a Hurwitz polynomial for either $a_i > 0$ or $a_i < 0$, because

$$Bez\left[\sum_{i=1}^{T} a_i p_i^e(s), \sum_{j=1}^{T} a_j p_j^o(s)\right] > 0$$

△

*Lemma A*2.[13] *Let* $a(s)$, $b(s)$ *be polynomials with* max[deg $a(s)$, deg $b(s)] \leq n$. *Then:*

(*a*) *The Bezoutian* $Bez[a(s), b(s)]$ *is a symmetric matrix.*

(*b*) *The Bezoutian* $Bez[a(s), b(s)]$ *is linear in* $a(s)$ *and* $b(s)$.

(*c*) $Bez[a(s), b(s)] = -Bez[b(s), a(s)]$.

Note that $Bez[b(s), b(s)] = 0$ for any polynomial $b(s)$, by (c) in Lemma A2.

*Lemma A*3.[14] (*The Liénard-Chipart criterion*). *Let* $a(s) = a_n s^n + a_{n-1}s^{n-1} \cdots + a_0$ *be a real polynomial and let* $a(s) = a^e(s^2) + sa^o(s^2)$ *be its decomposition in even part and odd part. Then* $a(s)$ *is Hurwitz if and only if the coefficients of* $a^e(s)$ *have the same sign as* a_n, *and the Bezoutian* $Bez[a^e(s), a^o(s)] > 0$ *is positive definite.*

Proof of Proposition 1. The family F_{12} is simultaneously stabilized by the controller $c_{xy}(s)$ if the following polynomial is Hurwitz $\forall\ a, b, x, y \in R^+ - \{0\}$ or $\forall\ a, b, x, y \in R^- - \{0\}$

$$P_{ab}(s) = [aN_1(s) + bN_2(s)][xN_{c1}(s) + yN_{c2}(s)] + [aD_1(s) + bD_2(s)][xD_{c1}(s) + yD_{c2}(s)].$$

the above polynomial can be rewritten as

$$ax[N_1(s)N_{c1}(s) + D_1(s)D_{c1}(s)] + ay[N_1(s)N_{c2}(s) + D_1(s)D_{c2}(s)] + bx[N_2(s)N_{c1}(s) + D_2(s)D_{c1}(s)] + by[N_2(s)N_{c2}(s) + D_2(s)D_{c2}(s)]$$

since the polynomials $q_i^e(s^2) + sq_i^o(s^2)$ are Hurwitz for $i = 1, 2, 3, 4$. Then by Lemma A1, the polynomial $P_{ab}(s)$ is Hurwitz $\forall\ a, b, x, y \in R^+ - \{0\}$ or $\forall\ a, b, x, y \in R^- - \{0\}$. △

Proof of Proposition 2.

(1) the case $\forall\ a, b, u, v \in R^+ - \{0\}$ is proven. The F_2 family is simultaneously stabilized by each controller in the family of controllers $\bar{c}_{uv}(s)$ if the following rational function is a strict positive real function of zero relative degree [that is, SPR0 function, notice that a SPR0 function is a unit in $U(RH^\infty)$].

$$P_{abuv} = (an_1 + bn_2)[u(x_1 + r_1 d_1) + v(x_2 + r_2 d_2)] + (ad_1 + bd_2)[u(y_1 - r_1 n_1) + v(y_2 - r_2 n_2)]$$

$\forall\ a, b, u, v \in R^+ - \{0\}$ or $\forall\ a, b, u, v \in R^- - \{0\}$. This polynomial can be rewritten as:

$$P_{abuv} = au(n_1x_1 + d_1y_1) + av[n_1x_2 + d_1y_2 + r_2(n_1d_2 - d_1n_2)] + bu[n_2x_1 + d_2y_1 + r_1(n_2d_1 - d_2n_1)] + bv(n_2x_2 + d_2y_2)$$

combining the above equation with Eqs. 14 and 15

$$P_{abuv} = au + av[n_1x_2 + d_1y_2 + r_2(n_1d_2 - d_1n_2)] + bu[n_2x_1 + d_2y_1 + r_1(n_2d_1 - d_2n_1)] + bv.$$

now because Eqs. 16 and 17 in the Proposition 2 are SPR0 functions, positive constants are SPR0 functions and linear combination of this SPR0 function,[16] imply that P_{abuv} is an SPR0 function, and a unit, too $\forall\ a, b, u, v \in R^+ - \{0\}$.

(2) the case $\forall\ a, b, u, v \in R^- - \{0\}$ is similar, however, the set of SPR0 functions must be multiplied by (-1). △

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